Second nature
For a liberal naturalism of mathematics

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Abstract
The term "naturalism" has, over the course of the history of philosophy, taken on different and changing meanings, so we can apply it to a large number of philosophical areas, all having in common only an unspecified reference to the field of nature. Fortunately, contemporary naturalism has changed in recent years, with less erratic and ambiguous connotations, thus allowing for the possibility of identifying two distinct meanings: "scientific naturalism" and "liberal naturalism", respectively. In this paper, we shall demonstrate how the distinction between different types of naturalism may more or less positively affect the field of numbers and arithmetic properties.

Keywords
Naturalism, liberal naturalism, mathematical naturalism, Maddy scientific naturalism.

Introduction

The term "naturalism," over the course of the history of philosophy, has taken on different and changing meanings, so we can apply it to a large number of philosophical areas (the Ionian philosophers, Aristotle, the philosophy of Hume and Spinoza, nineteenth-century positivism, logical empiricism and pragmatism, to name just the most popular areas), all having in common only an unspecified reference to the field of nature. Fortunately, contemporary naturalism has changed in recent years, with less erratic and ambiguous connotations, thus allowing for the possibility of identifying some common traits in different fields of application (De Caro & Macarthur 2004). Generally, two distinct meanings can be identified, "scientific naturalism" (much better known) and "liberal naturalism" (less known, but which in recent years has had a rapid rise), respectively. Both of the two perspectives share what might be called the "constitutive theory" of naturalism, namely, the use of laws, explanations, and entities that are given in nature and therefore do not belong to the realm of the supernatural (religious beliefs, mysticism, demiurges, deities, and so on). In addition, both modern conceptions of naturalism agree that the natural sciences are the ideal model to which all other sciences must comply in order to be legitimated in their cognitive activity.

However, although both concepts make claims as to the value of natural science and the experimental data that can be derived from it, the two views are divided on the role to which philosophy should be assigned. In fact, according to the conception of scientific naturalism, which in its most radical form has been commonly traced back to Quine (but which has also been associated with the perspectives of the analytic philosophers such as Dennett and Churchland), philosophy is not an activity that arises from a point of view that is "external" to the natural sciences (as theorised by, among others, Aristotle, Descartes and Kant), but rather, philosophy is, in itself, a part of science: it arises as part of our system of the world, in continuity with the rest of science. In short, Quine argued for the need to abandon once and for all the "dream" of a Philosophia Prima, a philosophy that is more important than natural science: the Philosophia Prima must give way to the Scientia Prima.

In contrast, theorists of the liberalised conception of naturalism, though they also believe that scientific knowledge is fundamental to philosophy and that philosophical formulations must take into account the achievements of natural science, do not accept the "continuity" thesis of scientific naturalism because for those authors, philosophy differs from science in the method, object and purpose of the research. According to the theorists, only in this way can philosophy overcome the sharp division that exists in the scientific version of naturalism, between the actual phenomena of the physical world and those that relate to the different fields of human existence. This allows the recovery concepts such as normativity, intentionality and free will, which are hardly reducible to the physical world, thereby giving them the dignity of belonging to the domain of the natural world, away from any metaphysical contamination.

We shall see below how this distinction between different types of naturalism may affect more or less positively the field of numbers and arithmetic properties.

1. Scientific naturalism and the philosophy of mathematics

As we have said, for Quine and for all of the scientific naturalists, the intent and purpose of mathematical research is to seek solutions of science and philosophy together because they cannot be separated. In this sense, mathematics involves cognitive processing, precisely like the theoretical aspects of science. In the same vein, Quine asserted that, although not entirely faithful to the original spirit of mathematics, the leading figure in twentieth-century mathematical naturalism is P. Maddy. In fact, even referring to Quine in terms of the importance of the scientific method, which can also be associated with the pragmatic approach more than that of the mathematical community, but convinced of the value of the presumed ability of mathematicians to judge and control the construction of their theories, and thus contradicting one of the main theses of scientific naturalism, which only processes information in terms of that which is scientifically useful (and not according to the criteria that is defined within the community of mathematicians) as the only criterion of acceptability of a thesis. However, given the particular field of her research, Maddy was able to provide definitions for the entities that philosophers discuss, but which, at least *prima facie*, are not attributable to the entities that have been postulated by the natural sciences (in her specific case, the abstract entities of mathematics). This is the so-called "placement problem," otherwise understood as the problem of identifying the location of these entities in the natural world. To this specific issue, Maddy has responded over the years, first by supporting a form of mathematical Platonism (called "realism set theory"), and even going so far as to apply the "principle of indispensability" to justify the realism of mathematical entities by virtue of her argument that the objective existence of abstract entities is integral to the best explanation we have of the world (according to Quine's holistic network and the role that mathematics plays throughout). However, justifying Platonism requires that we make room for the faculty of mathematics, which, on the other hand, is criticised from the point of view of nature in values.

Maddy has attempted to object to this criticism by exposing the point of view that mathematical intuition is not only similar to sensory intuition, as Godel claimed, but rather, it is a faculty of perception, i.e., the perception of sets of medium-sized physical objects, whose formation can be detected in the brain (Maddy, 1990). In this sense, we arrive at the second strategy that has been adopted by Maddy, i.e., the reductionist strategy, according to which these properties are ontologically genuine, but only because they are ontologically identical to, or occurring according to, scientifically acceptable properties. In Realism in Mathematics, 1990, Maddy indicates that the discoveries that were made in those years came from neuroscience and experimental psychology in order to focus on the analogy between the insight of sets and the perception of objects (an idea was intuited by Godel, as mentioned above). Drawing on the findings of the neurophysiologist Donald Hebb (1980), who showed that neurons not only confine themselves to carrying out immediate perceptual activity but, on the contrary, they continue their mutual functional electrical stimulation well after the cessation of sensory stimulus, by which are formed "cell assemblies" (groups of neurons in the connection), Maddy identifies in these groups of neurons that maintain a connection with each other the neurophysiologic consideration of her idea of "physical object."

In other words, according to the author, in order to form a gathering of cells that are capable of grasping an object, for example, "delta," it will first require the gathering of the mobile phone that is capable of detecting the angles. Then, these will give rise to the gatherings that are able of capture a certain type of triangle from a certain perspective, and then a number of gatherings of different perspectives give rise to a perspective that integrates the various perspectives as described above, thereby referring them to a single object. Maddy is convinced that these observations fully validate, from the neurophysiologic point of view, Piaget's theories concerning the formation of the concept of the object by children. In fact, she also writes in the same work: "This expectation is substantiated by the experiments of Jean Piaget and his colleagues. The child's ability to acquire perceptual beliefs about physical objects, as judged from behaviour, develops between the ages of one and eighteen months. At the beginning of this period, the child's world is a welter of isolated incidents" (Maddy, 1990: 54). Therefore, according to the author, the same neurons are the ones that are set in motion by the continuing perception of the object, although from different perspectives by which it is perceived, which are continually challenged to keep their electricity on each other and by then generating a cell that serves as a gathering "object detector." From these considerations, especially based on Piaget's experiments of seriation and commissioning, Maddy suggests a similar development with regard to the formation of the assembly concept. In fact, she goes on to say: "In this way, even an extremely complicated September would have a spatial-temporal location, as long as it has things in the physical ITS transitive closure. And any number of different sets would be located in the examination place, for example, the set of the set of three eggs and the two set of hands is located in. The same place as the set of the set of two eggs and the set of the egg and the other two hands." (Maddy, 1990: 59).

Therefore, in Realism in Mathematics, Maddy is convinced of the value of Piaget's experiments and his idea that there is a relationship between a general intelligence structure and the evolution of mathematical competence. However, in the last 25 years, the Piaget model has been questioned for evidence of numerical capacities in animals and children. Many works have, in fact, shown that not only animals and children are able to represent numbers crudely, but that this ability summons brain structures that are similar among species. Furthermore, numerical experiments on adult cognition have highlighted the important role played by nonverbal processes and have shown how logic is not a primordial and primary aspect of numerical representation: mathematical ability, albeit an approximate one, seems be present in children from the earliest days of life, constituting a sort of universal jurisdiction that mathematical neuroscientist, Stanislas Dehaene (2011), calls "number sense," which we share with other animal species. Dehaene further suggests that this instinct is the expression of the operation of a mental organ, a set of brain circuits that are also present in other species, which functions as an accumulator, i.e., as a type of counter that allows us to approximately perceive, store and compare numerical quantities. Numerous studies with brain imaging techniques have in fact shown the role of a part of the parietal cortex, the intraparietal sulcus (more precisely, the horizontal part of the parietal lobe, the "bilateral horizontal segment of intraparietal sulcus," cfr. Dehaene et al. 2003), which is active in those tasks that appeal to these approximate representations.

In her 2007 book, Second Philosophy, Maddy (2007) acknowledges that neuroscientific research has been enriched by new discoveries, having become gradually more and more precise in detail with regard to the research on the nature of mathematical entities and that the field has improved decisively. Through the arguments of Realism in Mathematics, Maddy claims that, aside from the neurophysiologic findings we have today (especially those that are derived through the use of PET and fMRI), things are very different from those of 1980, when she addressed the work of Hebb. For these reasons, in Second Philosophy she refers specifically to Dehaene and Spelke and their numerical experiments that were related to cognition. However, even if the outcome of these experiments is unanimously certain, the fact remains that in terms of the latter interpretations, Maddy turns out to be quite controversial because, once again, she finds a way to make this material support the role that the theory of sets plays in mathematics. The result is that Maddy manages to bring out certain cognitive invariants that, according to her Platonist interpretation of set theory, correspond to the elementary properties of the objects of such theory. However, this is not enough to send us to some plausible epistemology for mathematics because the role upon which set theory in mathematics depends is held by the whole theory. Furthermore, as is shown by Parsons (2007), assuming that it is permissible to draw a conclusion from the description of the phenomena of perception that would be at the base of our elementary numbers, there remains the problem that both of the mathematical theories that are applied in this description, both those used in the neural and psychological theories, can be formulated, and, moreover, they have been quietly made, without invoking set theory in any way: so, "It is just not plausible that the formulation in terms of set theory reflects the nature of things to that the degree Maddy's view presupposes" (Parsons, 2007: 211). The problem, as evidenced by Parsons, seems ultimately to depend on whether the transition from elementary mathematical beliefs to empirically based processes that govern our mathematical theories cannot in turn be justified empirically, although the transition is clearly crucial in the building of mathematics itself. It is on this crucial point that Maddy's scientific naturalism has failed. However, has naturalism itself failed?

2. The liberal naturalism of mathematical entities

As we have said, the scientific naturalism of natural science constitutes the model to which all other sciences and philosophical reflection must comply in order to be legitimised in their cognitive activities. But, despite being a legitimate criterion in its principles, it proves to have great limitations in the face of mathematical concepts, such as numbers, time, and so on. For some authors, the irreducibility of these objects and the natural horizon that is based on the apparent intractability of natural science would suggest the need to eliminate them from the philosophical vocabulary and to replace them with scientific terms and concepts that provide greater consistency in the material plane. This is the view, for example, of Hartry Field with respect to mathematical properties. According to the author, in abstract classes, math does not exist, and therefore, the truth value of mathematical statements is identical to that of sentences like "Oliver Twist lived in London," that is, they are irrevocably false.

Fortunately, in recent years, a type of naturalism has become increasingly popular that is less radical than the scientific modes proposed by Quine-Maddy or than the eliminativism by Field, showing that there are other ways in which naturalists can go beyond the reductionist model of scientific naturalism. In this proposal, the key appears to be compatible (rather than continuous) between philosophy and science, which forcefully leads to the anti-reductionist focusing on themes of normativity. Scientific naturalism is not in fact able to provide an account for the inadequate explanation of the constitutive features of human nature. However, how can we reconcile the normative level with the causal (which is typical of the natural sciences)? This question comes in response to John McDowell, whose proposal is emblematic of the position that has been taken by mediating liberalised naturalists. According to McDowell, the specificity of human beings is unique because they come with a "second nature" (De Caro & Macarthur 2010). Referring to the notion of a "space of reasons" by Wilfrid Sellars, McDowell argues that the best way to explain some features of human behaviour is to refer not only to the "causes" that govern bodily movement but also, especially, the "reasons" for human actions: "reasons." However, these should not be considered to be abstract entities that are independent of human experience but, in contrast, they are an integral part of our nature (they are, in fact, our "second nature"). In this case, the liberal naturalism of McDowell meets the first requirement (which we might call ontological) on which scientific naturalism, as opposed to the difficulties presented by Maddy, namely the investigation into the nature of the explanations of all types of entities that are required by paragraph of this explanation, without a priori constraints: in this way we will not have any difficulty in accepting the existence of entities such as morals, as the modal or intentional (and the truth or falsity of the corresponding ratings), provided that these entities are essential to take into account the important aspects of our thinking, which means that no explanations can include supernatural entities that violate the laws of nature. In the case of mathematical entities, then, we must not commit a "misrepresentation," and proceed to privilege the real, once and for all, which is true only according to our beliefs and symbolic mathematics. To clarify the issue on mathematical entities, we need two different notions of existence, and liberal naturalism has no difficulty in explaining both notions. We borrow the dual notion of existence from a philosopher of language, Aldo Bonomi, who distinguishes between ***r***-esistance and ***l***-existence: "The ***r***-existence is, in the terminology above, the existence-in the ordinary sense. ***L*** - here it means to belong to a certain domain of interpretation. It is an existence that has a linguistic nature in the sense that objects exist-that owe their identity to linguistic criteria. All you need to state that something exists is that you can find that object in the logical space of discourse in which it appears" (Perconti, 2003: 10). Therefore, liberal naturalism as defined by McDowell, but also all by others who are inspired by this form of naturalism, has no difficulty in accepting conceptual analysis (and here we come to a second requirement, which is that of methodology) as a method that is a legitimate investigation unless it represents a fruitful way to explain certain phenomena, as long as this method can be proved to be incompatible with the investigations of the natural sciences, for example, neuroscientific investigations. If this is true, then normativity is not incompatible with a descriptive and causal investigation: that is to say, logically at least, that normativity can be compatible with descriptive and causal investigations.

However, does this hold for all knowledge? Let's review an example, taken from Pascal Engel (2001), in a field that is close to mathematics, which is that of decision theory. According to decision theory, in rational choice, "Bayesian" rational agents obey a minimal standard, which is that of maximising expected utility. The normative theory of rational choice under this formula and the principles of choice that flow from it. The descriptive part of the theory has the task of determining whether in fact these agents follow rules. As has been highlighted by several experimental psychologists, in certain circumstances, the agents do not follow the normative theory as theorised, which gives rise to certain paradoxes (one of the most famous is that of Allais), in which the agents will systematically move away from maximising their usefulness. Decision theorists may argue that this response is so irrational that it contains an error of reasoning or some factor that has influenced the response of causal agents. In this case, it therefore appears that interpretive understanding is not a causative factor. It is, as it were, opaque, and contains no reference to a rule, but rather it refers only to a psychological process that is responsible for the error, "but to say what it means give to understand that error, giving to understand why agents do" (Engel, 2001: 16).

While Engel's warning does not require us to accept how the cognitive epistemic is irreconcilable with the best available practices, one must nevertheless emphasise the example that he reported does not undermine liberal naturalism for several reasons. First, naturalism's apparent paradoxes includes errors, and decision theory can easily explain such shifts "for reasons" that are other than those prescribed by traditional the neoclassical theory of utility maximisation, for example, an agent can decide to give up today to try to maximise more tomorrow, or because they forego maximising, the agent thinks of acquiring social prestige, and so on. Therefore, we are not always true and just in advance of committing "errors," as theorised by experimental psychologists as well as by Engel. In secundis, a new paradigm is having more success in explaining economic changes in the context of a union between a formal explanation and a causal explanation of economic factors: the neuroeconomy that does not reject in toto the neoclassical explanation, but, rather, it tries to find neural correlates (Camerer, 2003). Finally, it should be noted that the liberal naturalist (perhaps McDowell can be excluded in this case) will have no difficulty in accepting a reduction or elimination, but only if this proves to be either impossible or epistemically fruitful.

If anything, the real problem of liberal naturalism, considered in the positive light of Engel, is that if we want to provide the description of not only a certain phenomenon but also the adequate explanation of why a certain thing happens, we should aim to answer the question of whether it is possible for humans (unlike other physical systems), to participate in a "second nature." In fact, when you engage in arguing that rational agents are natural systems, then you have almost groped duty to provide an answer to this question. To answer this question, however, we do not have to abandon liberal naturalism because all of the knowledge that is available to rational agents, including mathematics, is part of a natural process of adaptation. It is under such a process that mathematics has occupied an important place in the course of human evolutionary history as a decisive step towards the achievement of higher cognitive abilities, which has supported the formulation of hypotheses about the shapes of bodies that are present in the environment, as well as their position and their number. This has meant that humans have discovered more and more new properties of the environment that have led us to advance towards more appropriate behaviours and to have greater success. The need to formulate hypotheses derived, therefore, in the simplified view, from the signals that are provided by bodily sensory receptors, which were not sufficient and therefore required imbuing these signals with meaning, which, in itself, was ambiguous and susceptible to multiple interpretations.

The notion that mathematics is part of a natural process of adaptation is clear in arithmetic. As demonstrated by Stanislas Dehaene's experiments and those of other cognitive neuroscientists, the idea of numbers is not derived from our sensations (otherwise, children would have numerical concepts within a few days after birth, requiring only the ability to manipulate them) but we must assume that our brain has an innate ability that allows us to detect small numbers and that this ability is a product of evolution. Of course, arithmetic is not sufficient to develop these innate abilities, but we also need the ability to create systems of symbols, both spoken and written. Only by virtue of these additional skills may we appoint different infinite numbers, address continuous quantities of discrete things and invent the rules of arithmetic. The latter skills, however, are not the product of biological evolution, but rather they are the product of another type of evolution, a cultural one, which, unlike the former, is much faster and more accurate.

Therefore, the interrelationship of these considerations indicates that naturalism proves to be liberalised, even the best of all possible naturalisms, if only because the objectivist view of science is subjective and sees human beings as agents.

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